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Cooperative Learning and the Theoretical Nature of Mathematical Knowledge

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Cooperative learning and the theoretical nature of mathematical knowledge⁹

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1. Introduction

The contribution tries to show the necessary interplay and the connections between the following three fundamental elements of theoretical mathematical knowledge, ways of social and cooperative learning and intentions to improve the mathematical understanding of less able students.

Theoretical mathematical knowledge is conceived of as meaningful knowledge, not reduced to technical procedures and conventional rules, but forming its specific meaning from a holistic and systemic perspective. The particular problem of mathematical meaning is the paradox that it neither is inherent simply in the formal structure alone nor can be totally deduced from empirical and natural situations of reference, but is constituted in a specific manner as a relationship between empirical and formal levels (cf Steinbring 1991). In this respect the number concept for instance cannot be reduced simply to its algorithmic and operational procedures but has to be developed in its own theoretical "existence".

The ways of social and cooperative learning induce forms of communication and exchange exceeding the personal and individual frame and this requires to construct in the cooperation more abstract concepts to be able to communicate and at the same time socially accept these types of abstraction, to organize and make them in some way concise, abstractions which are not needed and even idiosyncratic for a single individual. This kind of cooperatively supported abstraction relates to the theoretical nature of mathematical knowledge.

The intentions to improve the mathematical understanding of less able students have to focus on the basic problem according to

which most teachers believe that their students first of all have to master automatically the operative mathematical procedures and then meaningful understanding will turn up spontaneously (cf. Steinbring 1989). This separation between mathematical procedure and meaning leads to a self reinforcing vicious circle: the exercise of rules requires to concentrate on technical instructions for the algorithmic procedure and to let aside the mathematical meaning which in turn induces an even stricter concentration on the external structure of mathematical operations for being able to take into consideration all its details and separating the meaning further, etc. The result of this vicious circle is that students have to learn basic procedures completely by heart without any referential mathematical meaning, and in this way they are often in big troubles with the search for the correct mathematical rule. This disappearance of mathematical meaning in the exercise of rote procedures can only be overcome by respecting the theoretical nature of mathematical knowledge.

2. Theoretical mathematical knowledge and subjective domains of experience: playing games

Situations of learning with cooperative structures seem to be very useful to support the development and the treatment of theoretical mathematical knowledge which is in some way abstracted from the concrete empirical context and in another way refers back to it by offering a changed perspective. Theoreticity of knowledge and abstraction from empirical properties and situated conditions are necessary in cases where communication exceeds the individual bounds and becomes inter-subjective and cognition goes beyond the observable surface. To act cooperatively does in this respect not simply mean that the participating students separately execute their procedures independently, but the cooperative arrangement corresponds to a system of activity reflecting the systemic structure of the mathematical knowledge in question. In this way cooperative activity and communication contribute to generalization of mathematical knowledge by negotiating and legitimizing the general and abstracted concepts of knowledge.

The concept of mathematical game seems to fulfil many of the requirements which are asked for in the manifold connections between the theoretical mathematical knowledge, the ways of social and cooperative learning and the intentions to improve the mathematical knowledge, the ways of social and cooperative learning and the intentions to improve the mathematical understanding of less able students. The epistemological dimension of the mathematical knowledge, the social dimension of cooperative learning and the personal/psychological dimension of less abilities must be represented in the arrangement of the
structure of the mathematical problem. Used in a proper way, the mathematical game could display relations between all these three dimensions.

The epistemological dimension:
A mathematical game is "defined" by a set of rules, expressing a kind of axiomatic structure corresponding to the mathematical rules and procedures which are used here. On the one hand, these rules simply could serve as instructions to be applied in a sequential manner like the mathematical procedures are traditionally used; or they could serve as an elementary explicit description, a model of the system of all game relations. In this way the concept of a mathematical game represents mathematical knowledge, knowledge relations and meta knowledge, that is reflecting about the aims and the status of theoretical knowledge.

The social dimension
Being able to play the game with others requires to cooperate, to participate, to communicate, to obey the accepted rules, etc. In this respect, there is much implicit knowledge and social behaviour when playing a game, consisting of a mixture of explicit structures and implicit conventions, presuppositions, self evidences, and socially negotiated ways of casting different rules (cf Solomon 1989). This mixture of explicit structure and implicit ways of knowing, sharing and understanding is supported by and requires the social dimension of cooperation.

The personal/psychological dimension
To be engaged in a game, to play with the aim of winning, to make coalitions, to cooperate etc. all this requires social-affective and personal-emotional attitudes towards this game situations. Especially less able students are asked in a game situation to question their self consciousness, and to take over a new role in the game and in this way better understanding the role they are normally playing in the context of the formal learning situation. In this way the game situation could help to better overcome individual mathematical disabilities cooperatively, and at the same time make more clear how personal abilities are embedded and dependent on the social frame of expectations, former experiences and the difficult task of being able to personally play another part in this social frame.

The following social functions in the triangle of the three dimensions of “epistemological, social and personal” aspects can be observed in everyday mathematics teaching:
- to be able to take a role, to play in a strange context, to stand for something else,
- to be able to practise detachment and being at the same time emotionally involved
- to be able to cope with the paradox of a close personal and relative distance
- to be able to make differences between the everyday school situation and the game to be played, not to take the game "serious" in the same way as "real life".
- to experience the game as a separate, simplified world, as a simulation, allowing for better understanding and mastering the "real" world demands.
- to recognize one's own role better and being able to play it more effectively with regard to the game and not to take one's role in an absolute and indecisive manner.

In particular for the less able students, these personal and emotional requirements are very difficult to fulfil, because they often feel much more being questioned with regard to their individual abilities and habits as compared to other students, when they are subjected to mathematical problems framed in social patterns and expectations of role behaviour.

The following epistemological functions in the triangle of "epistemological, social and personal" aspects occur in everyday mathematics teaching:
- to be able to explore the rule structure: applying the rules, discovering connections, working backwards (from the anticipated goal to the start of the problem), varying the rules,
- to be able to explore and interpret the "variabilities" in the rule structure, to conceive of the rules as "relationships",
- to be able to examine the completeness and the connected system structure of the rule set.

These epistemological functions must not simply be considered as directed towards the technical aspects of the mathematical knowledge but in the tension between the theoretical character/the meaning of the mathematical knowledge and the situated and social conditions for the knowledge and its understanding.

The main focus of the following analysis of a transcribed teaching episode will be on the relation between cooperative ways of learning, organized in the frame of a mathematical game, and the specific nature and meaning of the mathematical knowledge as it is constituted in this particular interaction. Here we meet the central problem of developing theoretical mathematical knowledge and providing it with specific meaning. The following dichotomy is essential: the knowledge must be represented in some palpable frame, it must be given through a situation, the knowledge depends on the situation in which it is presented; intentions solve the mathematical problem given with the situation require to "abstract" some ideas and principles form the concrete situation and to generalize mathematical concepts. This interplay between the situatedness of the knowledge's meaning and the situation-
independent formal structure of mathematical knowledge is a crucial and a complex one.

On the one hand, students need their (personal) domains of subjective experiences (Bauersfeld 1988) for being able to equip the knowledge with meaning and references, on the other hand, communication and cooperation require to generalize and to abstract to a certain degree the knowledge from its proper situation. For coping with this difficult relation for mathematical knowledge between situation dependencies and necessities of abstraction the mathematical game could offer a fruitful frame which does not reduce or simply destroy the relation in question but makes it possible to use the concrete situation and the abstracted, general structure in a complementary way.

3. The reality of the math classroom: Analyzing a teaching episode

In this lesson, students have begun to play the game "Take 33" taken from the teaching material. At the blackboard, the teacher has already prepared the four rules (see annex), and she plays one trial run of the game with the students. In the course of the lesson, the students’ task is to play the game with the starting number 33 several times in partner work. After the end of partner work, the attempt is to have a common evaluation of individual games and game situations in an overall classroom discussion. The following analysis focuses on the teacher student-interaction starting after the partner work; this shall give some insights into the type of cooperation which has taken place during the playing of the game.

3.1 Description of the teaching episode

The episode "Number Games" can be structured into six sub sections:

1. Phase (907 to 954) Gerd and Andy explain the course of their game
2. Phase (955 to 994) Sascha C. and Hammit explain the course of their game
3. Phase (995 to 1016) Are there still other courses of the game?
4. Phase (1017 to 1032) Frank R. and Sascha T. explain the course of their game
5. Phase (1033 to 1113) Question: "Can you always win?"
6. Phase (1114 to 1132) Closing the lesson: homework

During the first four phases, three concrete runs of the game are presented and written into the prepared scheme on the blackboard.
The work in the class proves to be an arithmetical exercise, resulting in the following three "solutions"; Gerd and Andy have subtracted 4 times 5 and 4 times 3 (3 and 5 alternately), Sascha C. and Hammit have subtracted also 4 times 5 and 4 times 3 (3 and 5 in a different order), and Frank R. and Sascha T. have subtracted 7 times 3 and 2 times 5 (see the development of the blackboard images from 17 to 24 in the annex). This kind of arithmetical evaluation of some games played provides the background for the fifth phase, where demands for a possible winning strategy are raised; this episode is described in more detail (compare the appended transcript).

5. Phase (1033 to 1113) Question: "Can you always win?"
In this phase, justifications and strategies are searched for. The point is not only to demonstrate the concrete course of the game, but also to state certain evaluations and justifications. In this phase, justifications are presented essentially by three students, and accordingly the phase is subdivided into 3 sub phases.

5.1 Sub-Phase (1044 to 1057) Gerd’s explanation: Chance
Essential, Gerd voices the opinion that whether one wins or not depends on chance. For the partner can play as well and may choose freely which number he would like to subtract. Gerd says, for example: "...that won’t work then. That will hardly work. One can only win by chance, or not." (1046,7); or: "I cannot know what he will write next.... whether he (has) a 3 or a 5. And if I tell him, he won’t write it anyway." (1050, 53). Other students point out, too, that there is chance here and nobody knows who will win.

5.2 Sub-Phase (1058 yo 108) Frank K. refuses to give a justification for a game strategy
The teacher explicitly calls the student Frank K., as she thinks he sees an opportunity for a winning strategy. The student refuses to take part in the classroom discussion. In the interaction with other students, the teacher provokes claiming that she could always win. "Why not? Well, I think, I could always win if I play with you." (1068). Some students would like to test whether this claim is really true. In this connection, Frank K. points out explicitly that he has won against the teacher. "Yes, you have, you have lost against me here." (1073). The teacher plays this down by stating that her intention was to allow the student to win, too, just for once, a statement which provokes an ironical reaction of the other students. Upon this, the teacher justifies herself once more pointing out that Frank did not understand the game at the beginning; other students respond by saying: "I didn’t understand anything either." (1081).
5.3 Sub-Phase (1082 to 1113) Detlef proposes a solution
At the beginning of this sub-phase, the teacher again poses the question: "Can you always win?" (1082). Students' responses prompt the teacher to repeat this question explicitly: "It stands here. Yes, you can always win. Or is there a possibility that the first, in the end, basically, always gets 1 or the second always 2?" (1086 - 88). Most of the students very probably do not understand the intended meaning of the question aiming at a possible winning strategy. They say that one player will win in any case, or they are unable to give an answer. Only the student Detlef has a first approach to a possible solution. He says: "...you want that he notes a 3, but he notes a 5. Then you can, if he does this, again compensate this." (1096,7). By referring to the numbers from the example, the student intentionally tries to formulate a general principle, that is the principle of compensating, i.e. of choosing numbers dependent on which numbers his partner has chosen.

The teacher considers this to be an answer to the objection of one student who, however, did not pay attention. Upon this Detlef repeats his proposal: "Well, if you now want that he notes a 3, and he doesn't do it, then you can compensate with the other number." (1102,3).

This still tentative and preliminary argument which has not yet described the entire strategy in detail is not considered to be sufficient by the student Gerd. To the proposal of compensation which he seemingly interprets too extensively, Gerd responds: "...you can't do it for.... there are only two numbers to choose from. Now you cannot another number." (1105,6). And, going further, Gerd strengthens this objection: "Otherwise, you could always win. Who has last turn, could always win. If there is, for example, a 9 noted, and you want to have a 1. If you can choose any number, then I would take 8. Then I would get 1." (1108-10). Gerd emphasizes again that only the numbers 3 and 5 can be chosen, and therefore Detlef's argument is not valid, what the teacher confirms, too.

The solution approach proposed by Detlef, involving the strategy of compensating is understood by Gerd in particular to generalize the possibility of compensation (inadmissibly) to the possibility of choosing any number for subtraction. For him, this becomes an objection against the solution strategy proposed by Detlef. In this way, reflecting and developing a winning strategy is not finally achieved, it remains open. The explanation "chance" or "luck" stands in opposition to a seemingly deterministic strategy. It is not seen that this combination of compensating and two pre-given numbers are sufficient to elaborate a strategy.

The sixth phase closes the lesson with the formulation of the homework (see blackboard image 25).
3.2 Analysis of the teaching episode across the six phases

Considering the teaching episode as a whole, two different modes of procedure can be identified: During the first four phases, the course taken by several games is discussed by all on the blackboard. In this discussion, the teacher also introduces a new symbolic notation, she writes down, how often the numbers 5 and 3 have been subtracted during the game. During these phases there is no hint at all to possible winning strategies, to possible alternative subtractions which might have resulted in a changed winning outcome towards the end of each game. The game played is noted as it went, and there is a final summary counting how often 3 and 5 have been subtracted.

That there is no idea or desire in any of the students to look for a strategy during this phase can be seen from their statements that the 3 and the 5 have obviously been subtracted simply schematically in the first game.

In contrast to that, there is an attempt in the 5th phase to understand the question: "Can you always win?" and perhaps even to answer it. For many students the answer is that it is a case of chance or luck, or that nobody can determine who will win at the end. The teachers' claim that she could always win is not taken serious by the students, especially as she does not want to play and she has already lost against Frank K. The students' statements and in particular Frank's reaction show that they do not believe that the teacher knows a safe strategy.

Only one student, Detlef, takes the first step to a solution; he has the idea of compensating between the numbers 3 and 5. This idea, however, is not taken up and elaborated, that it becomes comprehensible (for instance by referring to the course of the game at the end of play, what is seemingly less complex and easier to foresee, anticipate and compare alternative outcomes).

The idea of compensating, in particular, is generalized by Gerd and interpreted inadmissibly as a possibility of compensating arbitrary numbers and not only 3 and 5. On this basis, Gerd constructs a counter argument against Detlef's idea: "Otherwise, you could always win." (1108), expressing that under such (extended) conditions of compensating one could always win. With this counter argument, the situation remains open, nobody is feeling obliged or in a position (at this point not even the teacher) to maintain and elaborate Detlef's argument. The lesson is closed with formulating the homework.

4. Some consequences

As different from similar teaching episodes, where the inquiry into justifications frequently results in funnel patterns of interaction (cf.
Bauersfeld 1978) and in reductions of mathematical meaning (cf. Steinbring 1989), which is always accompanied by a switch to the technical and operative level, such a form cannot be observed during this episode. A possible first solution approach contained in Detlef's proposal is invalidated by a counter argument; in this way the interaction remains open and undecided with regard to the search for a justified winning strategy. Most of the students continue to assume that the outcome of this game is unpredictable and depends on chance.

It is interesting in this episode, to investigate which conception of "game" the students have. Normally, one would assume, that every student wants to win and therefore is looking for a strategy, for different choices, and for some tactics. After all, there are written rules of the game concerning a win, a draw, etc. Something which might possibly provoke students' attitudes like being concerned or involved, or expressing emotional features as in other familiar competitive games, seems to remain outside the emotional sphere in this number game with its arithmetical operations. The game here seems to be a "game with numbers" and at the same time this conception of "game with numbers" seems to be linked to the idea that you will find here some arbitrary numerical rules not really allowing for a winning game. This habit towards the game "Take 33" is supported by Gerd's statements which have been repeated by others, simply trying to subtract 3 and 5 alternately, i.e. in some mathematically ordered manner. Other subtractions leading to the outcome of 1, may be chosen as well, but there is no conception for such a choice. And winning or not winning is no interesting question at the outset of the game. In addition to that, the teacher's way of noting the results reinforces the assumption that this is no ordinary game but a game with numbers. The factors "4 times 5", etc. written on the blackboard seem to represent a further arithmetical operation for rapidly counting how often each admitted number has been subtracted; the main focus is on correctly calculating and not on the rejection of how to choose adequate successions of numbers.

The factors written down by the teacher are not used comparatively either to clarify and to further develop possible ideas of students for winning strategies. The demand for the game courses showing other successions of 3 and 5 only seems to be interesting on the arithmetical level indicating that there have to be other arithmetical operations to be fulfilled in this game. And again these different runs of games are not contrasted for reflecting upon modified choices of numbers for "correcting" at least the three last numbers of a certain game to change the obtained outcome in a favorable one. The games played are only used as a kind of machine having produced a lot of numbers to be mathematically ordered. A concrete single run is not "generalized" by varying the choise of numbers for subtraction.
The fact that this "game with numbers" is an arbitrary methodical game, that there is no strategy for the students, even that this is perhaps no real game at all, but only a disguised form of mathematical operations with numbers, is also salient in phase 5. Regardless of the question: "Can you always win?", this is not interpreted in the sense of looking for a winning strategy. Gerd stresses at the beginning already, that it is a matter of chance, one does not know what the partner will choose and therefore it is impossible to predict the outcome of the game. The teacher's claim she could always win, meets with the students' refusal. And Detlef's proposal of compensation is exaggerated to the idea of compensation by taking arbitrary numbers, in this way constructing a (false) counter argument against a possible deterministic winning strategy.

Many students' statements show that they somehow feel exposed to this game, that someone will win in any case, that they complain that the partner has won again. It is observable that the students do not adopt an active and conscious strategy towards the game, and do not consider it possible and estimate this game of numbers in the math classroom as the familiar arithmetical procedures and operations they widely know already. One has to work according to some mystic rules and instructions, one writes numbers into schemes and reaches a result in this way. Additionally in this case, the result in some mysterious way declares one of the partners to be the winner.

The kind of cooperation taking place between two students when playing the game could be classified according to the four levels of cooperation as developed by Verstappen (Verstappen 1992,6). The interaction of two students reaches at most level 3: Second intermediate: \( S_1 \) and \( S_2 \) react incidentally to each other. The attainment of the fourth and highest level of cooperative learning requires that the students relate their collaboration to the theoretical nature of the mathematical knowledge inherent in this game situation. To be able to coordinate their activities the students have to develop an idea about arithmetical strategies for solving the problem, for instance working backwards from the result for reaching a winning start position.

Nowhere in this teaching interaction there is a discussion of the difficult combination between strategic-anticipating reasoning (perhaps possible to elaborate from the last turns of a play step by step) and the open-ended idea of compensation between two numbers. The main impression of this episode is that the students feel exposed to the game with numbers and also to this way of methodical procedure during the mathematics teaching. It is not they who play the game, but they are exposed to the pre-established rules, which do not admit insights or justifiable reasoning and other possible choices.

This teaching episode shows a double understanding of game:
first, as a form of a mathematically structured game with rules, procedures, experiences, strategic choices, etc., and second, the students' view of the game which they do not understand in its structure, but which they qualify as a game belonging to the familiar ordinary methodical games they have to play in mathematics teaching. The game, as intended in the teaching material, is not used to exercise an optimized mathematical computation while using the context of an intelligently structured game of competition. Rather, the interaction turns this game into one of the usual games occurring in daily classroom routine. It is thus deprived of its play character, becoming the known opaque and mystical rule machinery (cf. Seeger & Steinbring 1992) within which there is an obligation to do methodically correct computations, but where the outcome and the winner cannot be determined beforehand.

The central dilemma occurring in this short teaching episode between the strictly closed traditional patterns of interaction and the intended open-ended cooperative forms of work and discussion can be summarized in the following diagram:

Unlike in similar episodes, the interaction between teacher and students does not converge here rapidly to the algorithmic level of solution, despite the intentions of the teacher. This is due to the contradiction between the claim of the teacher. "I can always win!" and the students' claim that it depends on chance and luck who will win in the end. This contradiction is not solved and therefore in principle the two levels indicated in the above diagram remain parallel and are not reduced to the teacher's level. As the analysis of the fifth phase has shown, the students' level is not very profoundly elaborated, there are only some first hints and indications. But it seems obvious that the intentions of the teacher aiming directly at arithmetical operations prevent and even destroy theoretical mathematical meaning. Only if the still weak ideas of the students together with more cooperative discussions could become clearer, if the game really would be taken as a social and cooperative endeavour, then the mathematical knowledge in its cognitive and social relationships could become meaningful.
References


Appendix: The game "Take 33!"

Rule I: The first player subtracts 3 or 5 from the starting number 33 according to his own choice, writing down the residue into the next box!

Rule II: The second player subtracts 3 or 5 from the residue according to his own choice, writing down his own residue into the next box!

Rule III: Continue until the residue becomes smaller than 3!

Rule IV: If the residue is 1, the first player wins. If the residue is 2, the second player wins. If the residue is 0, the game ends with a draw.

Sequence of blackboard images

Single pictures are numbered by 17 etc.
Teaching episode of a mathematics lesson in grade 8.

Topic: Is it always possible to win? - Exploring the rules of a game with numbers.

1032 T.: Some haven't quite got it yet, what matters. If you will perhaps look the lower phrase. *(T points to the problem sheet)*
1033 S.: Can you always win?
1035 T.: Andy! The lower phrase thus is quite important. And some have already thought about it, I believe. Here Detlef and Michael, is my impression. Can you always win?
1038 Several S.: Nope!
1039 Several S. simultaneously: Yes!
1040 T.: Write down, how? but we can first once again.
1041 Michael intervenes (unintelligible)
1042 T.: Frank has said yes. But he should also say why, and how.
1043 Or Gerd. Did you want to say something?
1044 Gerd: Yes, I must know first what Andy wants to write down. And when we do it together, he can write down another number, and then you can think, and then he writes yet another number, that won't work then.
1047 That will hardly work. One can only win by chance, or not.
1048 T.: Well, Gerd says, because he doesn't know.
1050 Gerd: I cannot know what he will write next.
1051 Michael: *(unintelligible)*
1052 T.: Michael!
1053 Gerd: Whether he (has) a 3 or a 5. And if I tell him, he won't write it anyway.
1054 T.: No, no. He can write what he wants.
1055 Gerd: Yes!
1056 T.: Now, if you begin and think, I should like this and that.
1057 Gerd: If he always takes only 3, I would always (take) only 5.
1058 T.: Yes. But Frank thought there is a possibility that he will always win. How would that look like, Frank?
1059 Frank K.: No, I won't tell that.
1061 T.: Why not?
1062 Frank K.: I don't know anything.
1063 T.: Of course you know something.
1064 Frank K.: Sascha, you tell.
1065 Sascha C.: What?
1066 T.: Sascha, you mean you can always win?
1067 Sascha C.: *Nope!*
1068 T.: Why not? Well, I think, I could always win if I play with you.
1069 Gerd: Yes, o.k., let's try it.
1070 T.: Let's try it. I'm in favour of that, too.
1071 Sascha C.: *Why?*
1072 Gerd: Yes, go on, Sascha.
Frank K.: Yes, you have, you have lost against me here.
S.: Yes.
T.: I wanted to let you win once, too.
Several S. (ironically): Yep!
T.: But Frank did not understand at the beginning, the game, what it was about. That is why he wanted to work on the task sheet alone.
Michael or Detlef: Loud laughter.
T.: That's why I played with him.
Michael or Detlef: I didn't understand anything either.
T.: Michael, what do you mean? Can you always win?
Michael: Nope. (unintelligible)
T.: Yes, yes, alright, if you now mean, eh, whether Detlef can always win.
Yes.
T.: It stands here. Yes, you can always win. Or is there a possibility that the first, in the end, basically, always gets 1 or the second always 2?
Ina: (unintelligible)
Ina?
Ina: Nope! (Laughter)
S.: One always wins, shit.
T.: Yes, Detlef.
Detlef: (unintelligible)
Come on.
Detlef: ... you want that he notes a 3, but he notes a 5. Then you can, if he does this, again compensate this.
T.: Yes....yes, Gerd, that was an answer to your objection.
Gerd: Now I didn't listen.
T.: Yes, that always our difficulty here. Detlef, say it again.
Sascha C.: That goes.
Detlef: Well, if you now want that he notes a 3, and he doesn't do it, then you can compensate with the other number.
S.: Yes, that's what you can do.
Gerd: But if, you can't do it for... there are only two numbers to choose from. Now you cannot another number. (unintelligible)
T.: No, you can't.
Gerd: Otherwise, you could always win. Who has last turn, could always win. If there is, for example, a 9 noted, and you want to have a 1.
If you can choose any number, then I would take 8. Then I would get 1.
S. (ironically): Clever!
T.: Yes, and that's not possible, is it?
Gerd: That's the very problem. One can choose only two, 3 or 5.
T.: Yes!