Abstract—Synthetic aperture radar (SAR) techniques were originally developed to obtain high resolution images of scenes far away from the (airborne or spaceborne) imaging system. Data is collected over a large synthetic aperture to numerically obtain a higher resolution than that achievable by using the width of the antenna’s footprint at the range of the target to distinguish between distinct targets. However, even for short ranges, where a narrow beam produces high resolution due to only small beam widening on the short path, the application of wide beams in conjunction with numerical imaging techniques can be advantageous for detecting objects with radar cross sections that vary with the aspect angle. One of the degrees of freedom in the design of an imaging radar system is the antenna’s beamwidth. Issues concerning the choice of a proper beamwidth with respect to the estimation accuracy for targets with radar cross sections varying with the aspect angle will be addressed in the following.

I. INTRODUCTION

The azimuth resolution of a real aperture radar system is determined by the width of the antenna beam at the range of the target. For a certain divergence angle of the beam, the resolution decreases as the range to the target increases. In case of short ranges, the width of the antenna beam at the target might be made sufficiently small, so that for high azimuth resolution no additional signal processing might be needed. However, real world targets exhibit radar cross sections that vary with the aspect angle, which might yield additional problems. When a very narrow beam is used, the region of aspect angles under which the target is illuminated and the scattered signals are received by the radar system is very limited. Therefore, it is possible that an existing target is not detected by the radar system due to a disadvantageous aspect angle and its corresponding potentially low radar cross section. Illuminating targets from different aspect angles and thus increasing the amount of information obtained can be achieved by increasing the beamwidth and processing the received data into a radar image. In the following investigation, thermal system noise will not be considered. Only noise contributions due to the radar cross sections varying with the aspect angle will be taken into account.

II. MODEL OF THE TARGET

The type of target investigated in the following is an idealized one exhibiting an amplitude radar cross section consisting of $N$ statistically distributed specular reflections, modeled by Dirac pulses, each with equal phase and weight unity, where the probability for a reflexion to occur at a certain angle is equal for all angles. Therefore, the amplitude radar cross section $\rho_\vartheta(\vartheta)$ as a function of the aspect angle $\vartheta$ is

$$\rho_\vartheta(\vartheta) = \sum_{n=1}^{N} \delta(\vartheta - \vartheta_n),$$

where the values $\vartheta_n$ are uniformly distributed within the angle interval $[-\pi, \pi]$. Fig. 1 shows one of the possible realizations of the amplitude radar cross section for $N = 3$ randomly chosen angles.

III. IMAGING SETUP

A monostatic setup as depicted in fig. 2 will be considered in the following. A continuous signal at the frequency $\omega$ is transmitted by an antenna that is supposed to have a very small aperture. Its beamwidth is assumed to be $2\vartheta_0$. The antenna is directed into the positive $z$-direction. For the sake of simplicity, the antenna pattern is assumed to have a constant (non-zero) value for $-\vartheta_0 \leq \vartheta \leq \vartheta_0$ and to be 0 otherwise. The antenna is moved along the $x$-axis. An object at the location $(0, 0, z_t)$ is within the antenna’s beam for $-x_0 \leq x \leq x_0$, where

$$x = z_t \tan \vartheta$$

and especially

$$x_0 = z_t \tan \vartheta_0.$$  

It is assumed that the motion of the system is slow as compared to the velocity of the electromagnetic wave, $c$, so that the

![Fig. 1. A randomly generated $\rho_\vartheta(\vartheta)$ for $N = 3$](image-url)
Doppler effect can be neglected. The received signal is I/Q-mixed with the transmitted signal and phase and magnitude of the received signal are determined, so that for a target with amplitude radar cross section \( \rho_\theta(\vartheta) \) at the location \((0, 0, z_t)\) the result of this procedure is

\[
s_\vartheta(x) = \rho_\theta(\vartheta(x)) \cdot \frac{1}{d_\vartheta^2(x)} \cdot \exp \left\{ -\frac{2\omega d_\vartheta(x)}{c} \right\}, \tag{4}
\]

where

\[
d_\vartheta(x) = \sqrt{x^2 + z_t^2} \tag{5}
\]

is the distance between the antenna and the target. In terms of \( \vartheta \), the signal can be expressed as

\[
s_\vartheta(\vartheta) = \rho_\vartheta(\vartheta) \cdot \frac{1}{d_\vartheta^2(\vartheta)} \cdot \exp \left\{ -\frac{2\omega d_\vartheta(\vartheta)}{c} \right\}, \tag{6}
\]

with

\[
d_\vartheta(\vartheta) = \frac{z_t}{\cos \vartheta}. \tag{7}
\]

IV. IMAGE PROCESSING

Synthetic aperture radar techniques combine measurement results from various antenna positions to obtain a reflectivity image of the target scenery. One way to combine measurement results is to compensate the signals received at different angles of the target to obtain a reflectivity image from which the target location is estimated. In [1], this procedure is used for a circular path of the antenna around the scenery to be imaged. Analogously, signals obtained for measurements along a straight line can be compensated and integrated to obtain a reflectivity image. The compensation is accomplished by using a focusing operator

\[
\xi_\vartheta(\vartheta) = d_\vartheta^2(\vartheta) \cdot \exp \left\{ \frac{2\omega d_\vartheta(\vartheta)}{c} \right\}. \tag{8}
\]

The imaging result \( \iota(\vartheta_0) \) at the location corresponding to the true target location is

\[
\iota(\vartheta_0) = \int_{-\vartheta_0}^{\vartheta_0} s_\vartheta(\vartheta) \xi_\vartheta(\vartheta) d\vartheta = \int_{-\vartheta_0}^{\vartheta_0} \rho_\vartheta(\vartheta) d\vartheta. \tag{9}
\]

For \( \vartheta_0 = \pi \), i.e. for one complete circulation, the result is

\[
\iota(\pi) = \int_{-\pi}^{\pi} \rho_\vartheta(\vartheta) d\vartheta = 2\pi \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} \rho_\vartheta(\vartheta) d\vartheta = 2\pi \hat{\rho_\vartheta}, \tag{10}
\]

where \( \hat{\rho_\vartheta} \) is the mean value of \( \rho_\vartheta \) over all angles from \(-\pi\) to \(\pi\). Processing the signals received over an interval \([-\vartheta_0, \vartheta_0]\) in the proposed way and multiplying with a factor \( \pi/\vartheta_0 \) yields an estimate of \(2\pi\) times the mean value of the amplitude radar cross section. It is desirable to obtain an estimate that is close to the real value. A measure quantifying the inaccuracy \( \Delta_{rel} \) of the estimation is the mean square error normalized to the power of the desired signal

\[
\Delta_{rel}(\vartheta_0) = \frac{\mathbb{E}\left\{ \left( \iota(\vartheta_0) - \frac{\vartheta_0}{\pi} \iota(\pi) \right)^2 \right\}}{\left( \frac{\vartheta_0}{\pi} \iota(\pi) \right)^2}. \tag{11}
\]

Its reciprocal value can be interpreted as a signal-to-noise ratio (SNR), quantifying the accuracy of the estimation. The SNR can be written as

\[
\text{SNR}(\vartheta_0) = \frac{1}{\Delta_{rel}(\vartheta_0)}. \tag{12}
\]

Since all of the Dirac pulses forming the amplitude radar cross section exhibit weight unity, the integration of \( \rho_\vartheta \) over a certain interval of angles yields the number of Dirac pulses within the interval \([-\vartheta_0, \vartheta_0]\). Therefore, \( \iota(\vartheta_0) \) is a random variable for randomly chosen targets and can have integer values from 0 to \(N\). For a certain value of \( \vartheta_0 \), the probability for exactly \(n\) Dirac pulses being in the interval \([-\vartheta_0, \vartheta_0]\) is

\[
P\left( n \mid \vartheta_0 \right) = \binom{N}{n} \left( \frac{\vartheta_0}{\pi} \iota(\pi) \right)^n \left( 1 - \frac{\vartheta_0}{\pi} \iota(\pi) \right)^{N-n}, \tag{13}
\]

where

\[
p\left( \vartheta_0 \right) = \frac{\vartheta_0}{\pi}. \tag{14}
\]

is the probability for an angle randomly chosen out of \([\pi, \pi]\) to be in \([-\vartheta_0, \vartheta_0]\). The expected value of \( \iota(\vartheta_0) \) is (according to [2])

\[
\mathbb{E} \{ \iota(\vartheta_0) \} = \frac{\vartheta_0}{\pi} \iota(\pi) \tag{15}
\]

The SNR can therefore be written as

\[
\text{SNR}(\vartheta_0) = \frac{\mathbb{E}^2 \{ \iota(\vartheta_0) \}}{\mathbb{E} \{ \left( \iota(\vartheta_0) - \mathbb{E} \{ \iota(\vartheta_0) \} \right)^2 \}}, \tag{16}
\]

where the denominator is by its definition the variance \( \sigma^2 \) of the random variable \( \iota(\vartheta_0) \) given by (according to [2])

\[
\sigma^2(\vartheta_0) = \frac{N p(\vartheta_0) (1 - p(\vartheta_0))}{N p(\vartheta_0) (1 - p(\vartheta_0))} \tag{17}
\]

Using (15) and (17), the SNR can finally be reformulated as

\[
\text{SNR}(\vartheta_0) = \frac{N}{\vartheta_0 - 1}. \tag{18}
\]
For one complete circulation, i.e., $\theta_0 \to \pi$, the SNR tends to $\infty$, which means that the estimate is correct. In case the antenna is moved along the $x$-axis, the target can only be viewed at angles between $-\pi/2$ and $\pi/2$. The maximum possible value for $\theta_0$ is therefore $\pi/2$. That means that the maximum achievable SNR for a straight path is

$$SNR_{\text{max}} = SNR \left( \frac{\pi}{2} \right) = N.$$  \hfill (19)

Using an antenna with a half aperture angle smaller than 90° yields a suboptimal SNR. The degradation of the SNR with respect to $SNR_{\text{max}}$,

$$\Delta SNR = \frac{SNR(\theta_0)}{SNR_{\text{max}}} = \frac{1}{\tan^2 \frac{\theta_0}{2}},$$  \hfill (20)

is independent of $N$. The antenna’s beamwidth can be chosen in such a way that the degradation $\Delta SNR$ is within a certain limit. A half aperture angle $\theta_0 = 60^\circ$, for example, yields an SNR 3 dB below the maximum SNR achievable on a straight path.

V. SIMULATION

Simulations have been conducted for an antenna moved along the $x$-axis. Care has to be taken at the change of variables from the integration over the aspect angle to an integration over the $x$-coordinate. The integration

$$t_x(x_0) = \int_{-x_0}^{x_0} \rho_x(x) \cdot \frac{1}{z_1 \left( 1 + \frac{x^2}{z_1^2} \right)} \, dx$$  \hfill (21)

provides an unweighted estimate of $2 \cdot \rho_0 \cdot \arctan(x_0/z_1)$ for a target at $(0,0,z_1)$. The amplitude radar cross section as a function of $x$ instead of $\theta$ then is

$$\rho_x(x) = \sum_{n'} \delta \left( x - z_1 \tan \theta_{n'} \right) \cdot z_1 \left( 1 + \tan^2 \theta_{n'} \right),$$  \hfill (22)

where $n'$ are the indices of only those reflexions which are within $[-\pi/2, \pi/2]$ and therefore visible from locations on the $x$-axis. The simulations have been done as follows. For each number of reflexions $N$, 10,000 targets were generated randomly. For each of those targets, the signals received along the $x$-axis and the imaging algorithm for a target at a distance $z_1 = 1\, \text{m}$ were simulated for various half aperture angles between 1° and 89°. For each of those angles, the square error divided by the square of the desired value was determined and the mean value was calculated over all targets with equal number of reflexions. The SNR is the reciprocal value of this mean value.

Fig. 3 shows the SNR for various half aperture angles $\theta_0$ for targets with 1, 2, 5, and 10 specular reflexions, both predicted by (18) and obtained by simulation, on a logarithmic scale. The agreement of both calculation and simulation lends evidence to the analytic derivation.

VI. CONCLUSION

Synthetic aperture radar imaging using very narrow antenna beams exhibits the disadvantage that estimates of the mean amplitude radar cross section might be poor for targets whose radar cross sections vary with the aspect angle. The usage of wide antenna beams and appropriate signal processing can help improve the estimation performance. It has been proven analytically and substantiated numerically that the signal-to-noise ratio as defined above increases with increasing beamwidth. For antenna movement along a straight line, the maximum interval of aspect angles is limited to 180°. Therefore, the maximum achievable signal-to-noise ratio is limited. The maximum allowable degradation of the signal-to-noise ratio as compared to the maximum achievable signal-to-noise ratio determines the necessary beamwidth, which can be calculated using the formulas derived above.

REFERENCES
