Nonlinear elliptic-parabolic
integro-differential equations with $L^1$-data:
existence, uniqueness, asymptotics

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Abstract

The work is concerned with the existence and the qualitative behavior of solutions of certain nonlinear Volterra integro-differential equations. Both abstract equations with accretive operators and concrete equations of elliptic-parabolic type are considered. Equations of this type arise in the theory of heat flow in materials with memory and diffusion of fluids in porous media with memory.

Existence and uniqueness of entropy solutions is obtained for a concrete history dependent elliptic-parabolic initial boundary value problem for general integrable data.

Regularity properties of solutions are considered. In particular, existence of strong solutions is shown if the state space is a Banach space without the Radon-Nikodym property.

The asymptotic behavior of solutions is investigated. Conditions on the data are obtained such that the solution is asymptotically almost periodic or weakly almost periodic in the sense of Eberlein.

Keywords and phrases: entropy solutions, renormalized solutions, $m$-completely accretive operators, almost periodicity.

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